Unit 3 Lesson Summaries

Lesson 1 Summary

There are many real-world situations in which something keeps happening at the same rate. For example:

- a bus stop that is serviced by 4 buses per hour
- a washing machine that takes 45 minutes per load of laundry
- a school cafeteria that serves 15 students per minute

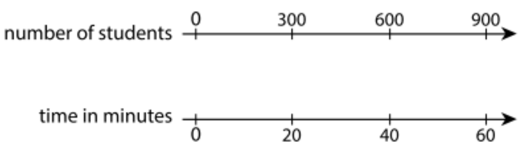
In situations like these, we can use equivalent ratios to predict how long it will take for something to happen some number of times, or how many times it will happen in a particular length of time.

- How long will it take the school cafeteria to serve 600 students?

  The table shows that it will take the cafeteria 40 minutes to serve 600 students.

number of students	time in minutes
15	1
60	4
600	40

- How many students can the cafeteria serve in 1 hour?



The double number line shows that the cafeteria can serve 900 students in 1 hour.

Lesson 2 Summary

We can use everyday objects to estimate standard units of measurement.

For units of length:

- 1 millimeter is about the thickness of a dime.
- 1 centimeter is about the width of a pinky finger.
- 1 inch is about the length from the tip of your thumb to the first knuckle.
- 1 foot is the length of a football.
- 1 yard is about the length of a baseball bat.
- 1 meter is about the length of a baseball bat and ball.
- 1 kilometer is about the distance someone walks in ten minutes.
- 1 mile is about the distance someone runs in ten minutes.

For units of volume:

- 1 milliliter is about the volume of a raindrop.
- 1 cup is about the volume of a school milk carton.
- 1 quart is about the volume of a large sports drink bottle.
- 1 liter is about the volume of a reusable water bottle.
- 1 gallon is about the volume of a large milk jug.

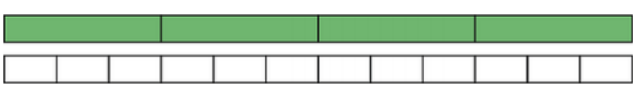
For units of weight and mass:

- 1 gram is about the mass of a raisin.
- 1 ounce is about the weight of a slice of bread.
- 1 pound is about the weight of a loaf of bread.
- 1 kilogram is about the mass of a textbook.
- 1 ton is about the weight of a small car.

Lesson 3 Summary

The size of the unit we use to measure something affects the measurement.

If we measure the same quantity with different units, it will take more of the smaller unit and fewer of the larger unit to express the measurement. For example, a room that measures 4 yards in length will measure 12 feet.



There are 3 feet in a yard, so one foot is $\frac{1}{3}$ of a yard.

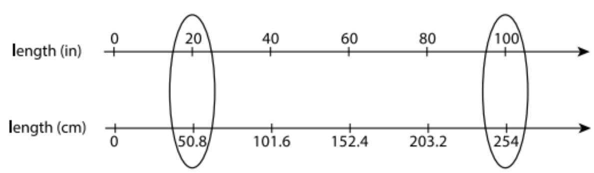
- It takes 3 times as many feet to measure the same length as it does with yards.
- It takes $\frac{1}{3}$ as many yards to measure the same length as it does with feet.

Lesson 4 Summary

When we measure something in two different units, the measurements form an equivalent ratio. We can reason with these equivalent ratios to convert measurements from one unit to another.

Suppose you cut off 20 inches of hair. Your Canadian friend asks how many centimeters of hair that was. Since 100 inches equal 254 centimeters, we can use equivalent ratios to find out how many centimeters equal 20 inches.

Using a double number line:



Using a table:

length (in)	length (cm)
100	254
1	2.54
20	50.8

One quick way to solve the problem is to start by finding out how many centimeters are in 1 inch. We can then multiply 2.54 and 20 to find that 20 inches equal 50.8 centimeters.

Lesson 5 Summary

Diego ran 3 kilometers in 20 minutes. Andre ran 2,550 meters in 17 minutes. Who ran faster? Since neither their distances nor their times are the same, we have two possible strategies:

- Find the time each person took to travel the *same distance*. The person who traveled that distance in less time is faster.
- Find the distance each person traveled in the *same time*. The person who traveled a longer distance in the same amount of time is faster.

  It is often helpful to compare distances traveled in *1 unit* of time (1 minute, for example), which means finding the speed such as meters per minute.

Let's compare Diego and Andre's speeds in meters per minute.

distance (meters)	time (minutes)
3,000	20
1,500	10
150	1

distance (meters)	time (minutes)
2,550	17
150	1

Both Diego and Andre ran 150 meters per minute, so they ran at the same speed.

Finding ratios that tell us how much of quantity A per 1 unit of quantity B is an efficient way to compare rates in different situations. Here are some familiar examples:

- Car speeds in *miles per hour*.
- Fruit and vegetable prices in *dollars per pound*.

Lesson 6 Summary

Suppose a farm lets us pick 2 pounds of blueberries for 5 dollars. We can say:

blueberries (pounds)	price (dollars)
2	5
1	$\frac{5}{2}$
$\frac{2}{5}$	1

- We get $\frac{2}{5}$ pound of blueberries per dollar.
- The blueberries cost $\frac{5}{2}$ dollars per pound.

The "cost per pound" and the "number of pounds per dollar" are the two *unit rates* for this situation.

A **unit rate** tells us how much of one quantity for 1 of the other quantity. Each of these numbers is useful in the right situation.

If we want to find out how much 8 pounds of blueberries will cost, it helps to know how much 1 pound of blueberries will cost.

blueberries (pounds)	price (dollars)
1	$\frac{5}{2}$
8	$8 \cdot \frac{5}{2}$

If we want to find out how many pounds we can buy for 10 dollars, it helps to know how many pounds we can buy for 1 dollar.

blueberries (pounds)	price (dollars)
$\frac{2}{5}$	1
$10 \cdot \frac{2}{5}$	10

Which unit rate is most useful depends on what question we want to answer, so be ready to find either one!

Lesson 7 Summary

The table shows different amounts of apples selling at the same rate, which means all of the ratios in the table are equivalent. In each case, we can find the *unit price* in dollars per pound by dividing the price by the number of pounds.

apples (pounds)	price (dollars)	unit price (dollars per pound)
4	10	$10 \div 4 = 2.50$
8	20	$20 \div 8 = 2.50$
20	50	$50 \div 20 = 2.50$

The unit price is always the same. Whether we buy 4 pounds of apples for 10 dollars or 8 pounds of apples for 20 dollars, the apples cost 2.50 dollars per pound.

We can also find the number of pounds of apples we can buy per dollar by dividing the number of pounds by the price.

apples (pounds)	price (dollars)	pounds per dollar
4	10	$4 \div 10 = 0.4$
8	20	$8 \div 20 = 0.4$
20	50	$20 \div 50 = 0.4$

The number of pounds we can buy for a dollar is the same as well! Whether we buy 4 pounds of apples for 10 dollars or 8 pounds of apples for 20 dollars, we are getting 0.4 pounds per dollar.

This is true in all contexts: when two ratios are equivalent, the two unit rates will always be equal.

quantity x	quantity y	unit rate 1	unit rate 2
a	b	$\frac{a}{b}$	$\frac{b}{a}$
$s \cdot a$	$s \cdot b$	$\frac{s \cdot a}{s \cdot b} = \frac{a}{b}$	$\frac{s \cdot b}{s \cdot a} = \frac{b}{a}$

Lesson 8 Summary

When two objects are each moving at a constant speed and their distance-to-time ratios are equivalent, we say that they are moving at the *same speed*. If their time-distance ratios are not equivalent, they are not moving at the same speed.

We describe **speed** in units of distance per unit of time, like *miles per hour* or *meters per second*.

- A snail that crawls 5 centimeters in 2 minutes is traveling at a rate of 2.5 centimeters per minute.
- A toddler that walks 9 feet in 6 seconds is traveling at a rate of 1.5 feet per second.
- A cyclist who bikes 20 kilometers in 2 hours is traveling at a rate of 10 kilometers per hour.

We can also use **pace** to describe distance and time. We measure pace in units such as *hours per mile* or *seconds per meter*.

- A snail that crawls 5 centimeters in 2 minutes has a pace of 0.4 minutes per centimeter.
- A toddler walking 9 feet in 6 seconds has a pace of $\frac{2}{3}$ seconds per foot.
- A cyclist who bikes 20 kilometers in 2 hours has a pace of 0.1 hours per kilometer.

Speed and pace are reciprocals. Both can be used to compare whether one object is moving faster or slower than another object.

- An object with the higher speed is *faster* than one with a lower speed because the former travels a greater distance in the same amount of time.
- An object with the greater pace is *slower* than one with a smaller pace because the former takes more time to travel the same distance.

Because speed is a *rate per 1 unit of time* for ratios that relate distance and time, we can multiply the amount of time traveled by the speed to find the distance traveled.

time (minutes)	distance (centimeters)
2	5
1	2.5
4	$4 \cdot (2.5)$

Lesson 9 Summary

Sometimes we can find and use more than one unit rate to solve a problem.

Suppose a grocery store is having a sale on shredded cheese. A small bag that holds 8 ounces is sold for \$2. A large bag that holds 2 kilograms is sold for \$16. How do you know which is a better deal?

Here are two different ways to solve this problem:

Compare dollars per kilogram.

- The large bag costs \$8 per kilogram, because $16 \div 2 = 8$.
- The small bag holds $\frac{1}{2}$ pound of cheese, because there are 16 ounces in 1 pound, and $8 \div 16 = \frac{1}{2}$.

  The small bag costs \$4 per pound, because $2 \div \frac{1}{2} = 4$. This is about \$8.80 per kilogram, because there are about 2.2 pounds in 1 kilogram, and $4.00 \cdot 2.2 = 8.80$.

The large bag is a better deal, because it costs less money for the same amount of cheese.

Compare ounces per dollar.

- With the small bag, we get 4 ounces per dollar, because $8 \div 2 = 4$.
- The large bag holds 2,000 grams of cheese. There are 1,000 grams in 1 kilogram, and $2 \cdot 1{,}000 = 2{,}000$. This means 125 grams per dollar, because $2{,}000 \div 16 = 125$.

  There are about 28.35 grams in 1 ounce, and $125 \div 28.35 \approx 4.4$, so this is about 4.4 ounces per dollar.

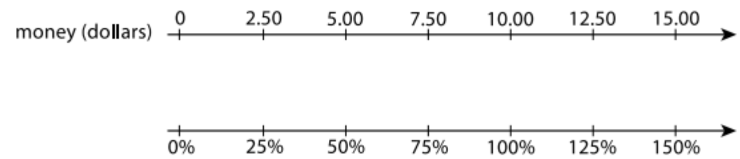
The large bag is a better deal, because you get more cheese for the same amount of money.

Another way to solve the problem would be to compare the unit prices of each bag in dollars per ounce. Try it!

Lesson 10 Summary

A **percentage** is a *rate per 100*.

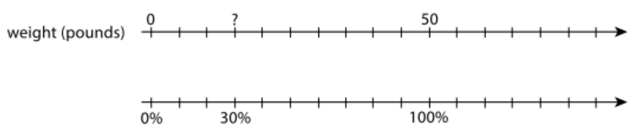
We can find percentages of \$10 using a double number line where 10 and 100% are aligned, as shown here:



Looking at the double number line, we can see that \$5.00 is 50% of \$10.00 and that \$12.50 is 125% of \$10.00.

Lesson 11 Summary

We can use a double number line to solve problems about percentages. For example, what is 30% of 50 pounds? We can draw a double number line like this:

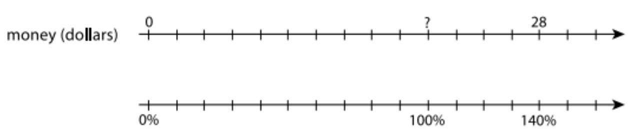


We divide the distance between 0% and 100% and the distance between 0 and 50 pounds into ten equal parts. We label the tick marks on the top line by counting by 5s ($50 \div 10 = 5$) and on the bottom line counting by 10% ($100 \div 10 = 10$). We can then see that 30% of 50 pounds is 15 pounds.

We can also use a table to solve this problem.

weight (pounds)	percentage
50	100
5	10
15	30

Suppose we know that 140% of an amount is \$28. What is 100% of that amount? Let's use a double number line to find out.



We divide the distance between 0% and 140% and that between \$0 and \$28 into fourteen equal intervals. We label the tick marks on the top line by counting by 2s and on the bottom line counting by 10%. We would then see that 100% is \$20.

Or we can use a table as shown.

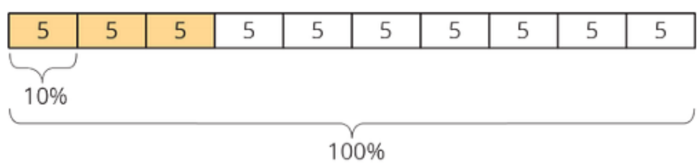
money (dollars)	percentage
28	140
2	110
20	100

Lesson 12 Summary

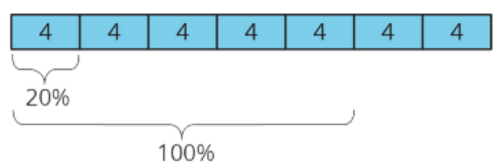
Tape diagrams can help us make sense of percentages.

Consider two problems that we solved earlier using double number lines and tables: "What is 30% of 50 pounds?" and "What is 100% of a number if 140% of it is 28?"

Here is a tape diagram that shows that 30% of 50 pounds is 15 pounds.

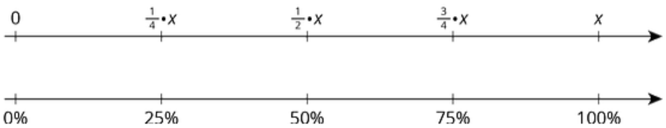


This diagram shows that if 140% of some number is 28, then that number must be 20.

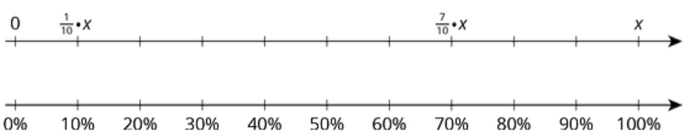


Lesson 13 Summary

Certain percentages are easy to think about in terms of fractions.



- 25% of a number is always $\frac{1}{4}$ of that number.
  For example, 25% of 40 liters is $\frac{1}{4} \cdot 40$ or 10 liters.
- 50% of a number is always $\frac{1}{2}$ of that number.
  For example, 50% of 82 kilometers $\frac{1}{2} \cdot 82$ or 41 kilometers.
- 75% of a number is always $\frac{3}{4}$ of that number.
  For example, 75% of 1 pound is $\frac{3}{4}$ pound.
- 10% of a number is always $\frac{1}{10}$ of that number.
  For example, 10% of 95 meters is 9.5 meters.
- We can also find multiples of 10% using tenths.
  For example, 70% of a number is always $\frac{7}{10}$ of that number, so 70% of 30 days is $\frac{7}{10} \cdot 30$ or 21 days.

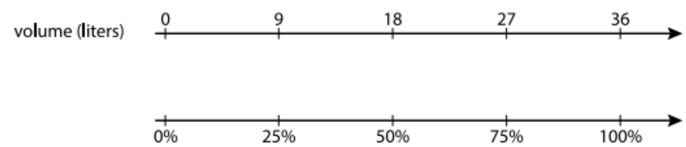


Lesson 14 Summary

A pot can hold 36 liters of water. What percentage of the pot is filled when it contains 9 liters of water?

Here are two different ways to solve this problem:

- Using a double number line:



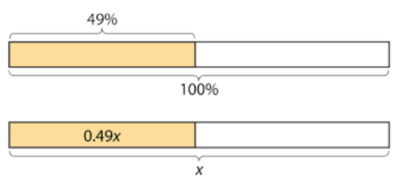
We can divide the distance between 0 and 36 into four equal intervals, so 9 is $\frac{1}{4}$ of 36, or 9 is 25% of 36.

- Using a table:

volume (liters)	percentage
36	100
9	25

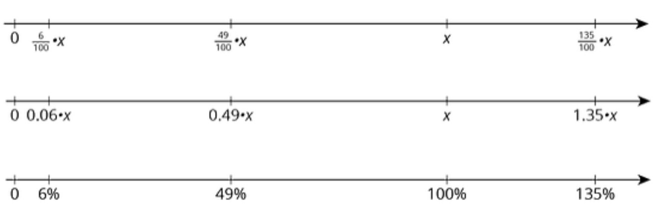
Lesson 15 Summary

To find 49% of a number, we can multiply the number by $\frac{49}{100}$ or 0.49.



To find 135% of a number, we can multiply the number by $\frac{135}{100}$ or 1.35.

To find 6% of a number, we can multiply the number by $\frac{6}{100}$ or 0.06.



In general, to find $P\%$ of x, we can multiply:

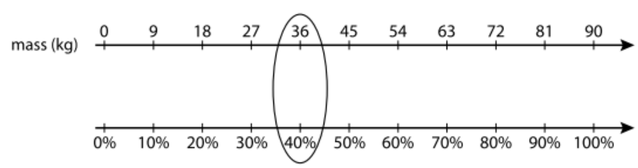
$$\frac{P}{100} \cdot x$$

Lesson 16 Summary

What percentage of 90 kg is 36 kg? One way to solve this problem is to first find what percentage 1 kg is of 90, and then multiply by 36.

mass (kg)	percentage
90	100
1	$\frac{1}{90} \cdot 100$
36	$\frac{36}{90} \cdot 100$

From the table we can see that 1 kg is $\left(\frac{1}{90} \cdot 100\right)\%$, so 36 kg is $\left(\frac{36}{90} \cdot 100\right)\%$ or 40% of 90. We can confirm this on a double number line:



In general, to find what percentage a number C is of another number B is to calculate $\frac{C}{B}$ of 100%. We can find do that by multiplying:

$$\frac{C}{B} \cdot 100$$

Suppose a school club has raised \$88 for a project but needs a total of \$160. What percentage of its goal has the club raised?

To find what percentage \$88 is of \$160, we find $\frac{88}{160}$ of 100% or $\frac{88}{160} \cdot 100$, which equals $\frac{11}{20} \cdot 100$ or 55. The club has raised 55% of its goal.